# PREVIOUS YEAR QUESTION BANK

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## STATICS & DYNAMICS

Q1. A particle slides down the arc of a smooth vertical circle of radius a being slightly displaced from rest at the highest point of the circle. Find the point where it will strike the horizontal plane thought the lowest point of the circle.

**(Year 1992)**
**(20 Marks)**

Q2. A particle is projected with a velocity whose horizontal and vertical components are respectively $u$ and $v$ from a given point in a medium whose resistance per unit mass is $K$ times the speed obtain the equation of the path and prove that if $K$ is small the horizontal range is approximately $\frac{2uv}{g} - \frac{8uv^2k}{3g}$.

**(Year 1992)**
**(20 Marks)**

Q3. A particle is moving with center acceleration $\mu(r^5 - c^4 r)$ being projected from an apse at a distance $c$ with a velocity $\sqrt{\left(\frac{2\mu}{3}\right)}c^3$ show that its path is the curve $x^4 + y^4 = c^4$

**(Year 1992)**
**(20 Marks)**

Q4. Defined central axis for a system of forces acting on a rigid body $A$ force $F$ acts along the axis of $x$ and another force $nF$ along a generator of the cylinder $x^2 + y^2 + a^2$ show that the central axis lies on the cylinder.

$$n^2(nx - z)^2 + (1 + n^2)^2 y^2 x = n^4 a^2$$

**(Year 1992)**
**(20 Marks)**

Q5. A semicircular of radius $a$ is immersed vertically with its diameter horizontal at a depth $b$. If the circumference be below the center, prove that the depth of center of pressure is $\frac{3\pi(a^2+4b^2)+32ab}{4(3b\pi+4)}$

**(Year 1992)**
**(20 Marks)**

Q6. Two equal rods, each of weight $wl$ and length $l$ are hinged together and placed astride smooth horizontal cylindrical peg of radius $r$, Then the lower ends are tied together by a string and the rods are left at the same inclination $\phi$ to the horizontal. Find the tension in the string and if the string is slack show that $\phi$ satisfies the equation $\tan^3\phi + \tan\phi = \frac{1}{2r}$

**(Year 1992)**
**(20 Marks)**

Q7. A particle is projected upward with a velocity $u$ in a medium whose resistance varies as the square of the velocity prove that it will return to the point of projection with velocity $v = \frac{uV}{\sqrt{u^2+V^2}}$ after a time $\frac{V}{g}\left(\tan^{-1}\frac{u}{V}\tanh^{-1}\frac{u}{V}\right)$ where $V$ is the terminal velocity.

**(Year 1993)**
**(20 Marks)**

Q8. A particle moves under a force $m\mu\{3au^4 - 2(a^2-b^2)u^5\}, a > b$ and is projected from an apse at a distance $a+b$ a with velocity $\sqrt{\frac{\mu}{a+b}}$ show that its orbit is $r = a + b\cos\theta$

**(Year 1993)**
**(20 Marks)**

Q9. A point executes simple harmonic motion such that in two of its positions the velocities are $u$ and $v$ and the corresponding accelerations are $\alpha$ and $\beta$ show that the distance between the position is $\frac{v^2-u^2}{\alpha-\beta}$

**(Year 1993)**
**(20 Marks)**

Q10. A semi circular lamina is completely immersed in water with its plane vertical, so that the extremity A of its bounded diameter is in the surface and the diameter make with the surface and angle $\alpha$ prove that if E be the center of pressure and $\phi$ the angle between $AE$ and the diameter $\tan\phi = \frac{3\pi+16\tan\alpha}{16\pi+15\pi\tan\alpha}$

**(Year 1993)**
**(20 Marks)**

Q11. A solid hemisphere is supported by a string to a fixed point on its rim and to point on a smooth vertical wall with which the curved surface of the sphere is in contact if $\theta$ and $\phi$ are the inclination of the sting and the plane base of the hemisphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$

**(Year 1993)**
**(20 Marks)**

Q12. The end links of a uniform chain slide along a fixed rough horizontal rod prove that the ratio of the maximum span to the length of the chain is $\mu \log(\frac{1+\sqrt{1+\mu^2}}{\mu})$ where $\mu$ is the coefficient of friction.

**(Year 1993)**
**(20 Marks)**

Q13. A particle of mass $m$ is projected vertical under gravity the resistance of the air being $mk$ time the velocity, show that the greatest height attained by the particle $\frac{V^2}{g}[\lambda - \log(1+\lambda)]$ is where $V$ is the terminal velocity of the particle and $\lambda V$ is the initial vertical velocity.

**(Year 1994)**
**(20 Marks)**

Q14. A gun is firing from the sea-level out to sea it is mounted in a battery $h$ meters high up and fired at the same elevation $\alpha$ show that the range is increased by $\frac{1}{2}\left[\left(1 + \frac{2gh}{u^2 \sin^2\alpha}\right)^{1/2} - 1\right]$ of itself u being the velocity of projectile.

**(Year 1994)**
**(20 Marks)**

Q15. If in a simple harmonic motion, the velocities at distances point $a, b, c$ from a fixed on the straight line which is not the center of force be $u, v, w$ respective show that the periodic time $T$ is given by

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

**(Year 1994)**
**(20 Marks)**

Q16. Show that the length of an endless chain which will hang over a circular pulley of radius $a$ so as to be in contact with two third of the circumference of the pulley is $a\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$

**(Year 1994)**
**(20 Marks)**

Q17. A smooth rod passes thought a smooth ring at the focus ellipse whose major axis is horizontal and rests with its lower end on the quadrant of the curve which is father removed from the focus Find its position of equilibrium and show that its length must at least be $\left(\frac{3a}{4}+\frac{a}{4}\sqrt{1+8e^2}\right)$ where $2a$ is the major axis and $e$, the eccentricity.

**(Year 1994)**
**(20 Marks)**

Q18. The height of a balloon is calculated from the barometric pressure reading (p) on the assumption that the pressure of the as varies $n^{th}$ as the density show that if the pressure actually varies as then power of the density there will be an error $h_0\left[\frac{n}{n-1}\left\{1-\left(\frac{p}{p_0}\right)^{\frac{n-1}{n}}\right\}-\log\frac{p}{p_0}\right]$ in the calculated height where $h_0$is the height of the homogeneous atmosphere and $p_0$ is the pressure at the surface of the earth.

**(Year 1994)**
**(20 Marks)**

Q19. If $t$ be the time which a projectile reaches a point $P$ in its path and $t$' the time from $P$ till it reaches the horizontal plane through the point of projection show that the height of $P$ above the horizontal plan is $\frac{1}{2}gtt'$ and the maximum height is $\frac{1}{8}g(t+t')^2$

**(Year 1995)**
**(20 Marks)**

Q20. A particle of mass m moved under a centre attractive force $m\mu\left(\frac{5}{r^3}+\frac{8c^2}{r^5}\right)$ and is projected from an apse at a distance with velocity $\sqrt[3]{\frac{\mu}{c}}$ prove that thee orbit is $r=c\cos\left(\frac{2\theta}{3}\right)$ and that it will arrive at the origin after a time $\frac{\pi}{8}\frac{c^2}{\sqrt{\mu}}$ .

**(Year 1995)**
**(20 Marks)**

Q21. Two bodies, of masses $M$ and $M'$ are attached to the lower end of an elastic string whose upper end is fixed and hang at rest $M$, falls off. show that the distance of $M$ from the upper end of the string at time $t$ is $a+b+c\cos\left(\sqrt{\frac{g}{b}}t\right)$ where a is the unstretched length of the string, and $b$ and $c$ the distances by which it would be stretched when supporting $M$ and $M'$ respectively.

**(Year 1995)**
**(20 Marks)**

Q22. A semi-ellipse bounded by its minor axis is just immersed in a liquid the density of which varies as the depth. If the minor axis be in the surface find the eccentricity in order that the focus may be the centre of pressure.

**(Year 1995)**
**(20 Marks)**

Q23. Two uniform rods $AB$ and $AC$ smoothly jointed at $A$ are in equilibrium in a vertical plane the ends $B$ and $C$ rest on a smooth horizontal plane and the middle points of $AB$ and $AC$ are connected by a string show that the tension of the string is $\frac{w}{(\tan B+\tan C)}$ where $W$ is the total weight of the rods and $B$ and $C$ are the inclination to the horizontal of the rods $AB$ and $AC$.

**(Year 1995)**
**(20 Marks)**

Q24. Prove that for the common catenary the radius of curvature at any point of the curve is equal to the length of the normal intercepted between the curve and the directrix.

**(Year 1995)**
**(20 Marks)**

Q25. A particle moves under gravity on a vertical circle, sliding down the convex side of smooth circular arc if its initial velocity is that due to a fall to the starting point from a height $h$ above the center, show that it will fly off the circle when at a height $\frac{2h}{3}$ above the center.

**(Year 1996)**
**(20 Marks)**

Q26. A stone is thrown at an angle $\alpha$ with the horizon from a point in an inclined plane whose inclination to the horizon $\beta$, the trajectory lying in the vertical plane containing the line of greatest slopes. show that if $\theta$ be the elevation of that point $bf$ the path which is most distance from the inclined plane then

$$2\tan\theta = \tan\alpha + \tan\beta.$$

**(Year 1996)**
**(20 Marks)**

Q27. One end of a light elastic string of natural length $a$ and modulus $2mg$ is attached to a fixed point 0 and the other to a particle of mass $m$. The particle is allowed to fall from the position of rest at 0. Find the greatest extension of the sting and show that the particle will reach 0 again after a time $(\pi + 2 - \tan^{-1} 2)\sqrt{\frac{2a}{g}}$

**(Year 1996)**
**(20 Marks)**

Q28. A hollow cone without weight closed and filled with a liquid is suspended from a point in the rim of its base If $\phi$ be the angle which the direction of the resultant pressure make with the vertical then show that $\cot\phi = \frac{28\cot\alpha+\cot^3\alpha}{48}, \alpha$ being the semi vertical angle of the cone.

**(Year 1996)**
**(20 Marks)**

Q29. A body consisting of a cone and a hemisphere on the same base rests on a rough horizontal … table. The hemisphere being in contact with the table Show that the greatest height of the cone, so that the equilibrium may be stable is $\sqrt{3}$ times the radius of the sphere.

**(Year 1996)**
**(20 Marks)**

Q30. A body of weight $W$ is placed on a rough inclined plane whose inclination of the horizon is $\alpha$ greater than then angle of friction $\lambda$.The body is supported by a force acting in a vertical plane through the line of greatest slope and make an angle $\theta$ with the inclined plane. Find the limits between which the force must lie.

**(Year 1996)**
**(20 Marks)**

Q31. A particle is projected along the inner side of a smooth circle of radius $a$ the velocity at the lowest point being $u$ show that $2ag < u^2 < 5ag$, the particle leaves the circle before arriving at the highest point what is the nature of the path after the particle leaves the circle?

**(Year 1997)**
**(20 Marks)**

Q32. A particle moves under a force $\{3au^4 - 2(a^2 - b^2)u^5\}, a > b$ and is projected from an apse at a distance $a + b$ with velocity $\sqrt{\frac{\mu}{a+b}}$ Find the orbit.

**(Year 1997)**
**(20 Marks)**

Q33. A shell bursts on contact with the ground and pieces from it fly in all direction with all velocities up to 80 units show that a man 100 unit away is danger for a time of $\frac{5}{2}\sqrt{2}$ units if g is assumed to be of 32 units.

**(Year 1997)**
**(20 Marks)**

Q34. A cylinder of wood (specific gravity $\frac{3}{4}$) of height h floats with its axis vertical in water and oil (specific gravity $\frac{1}{2}$) the length of the solid in contact with the oil is a $\left(<\frac{h}{2}\right)$ find how much of the wood is above the liquid Also find to what additional depth much oil be added so to cover the cylinder.

**(Year 1997)**
**(20 Marks)**

Q35. A solid frustum of a paraboloid of revolution of height hand latus rectum $4a$ rest with its vertex on the vertex of another paraboloid (inverted) of revolution whose latus rectum is $4b$. Show that equilibrium is state if $h < \frac{3ab}{a+b}$.

**(Year 1997)**
**(20 Marks)**

Q36. A heavy uniform chain rests on a rough cycloid whose axis vertical and vertex upwards, one end of the chain being at the vertex and the other at a cusp. If the equilibrium is limiting show that $(1+\mu^2)e^{\frac{\mu x}{2}}$

**(Year 1997)**
**(20 Marks)**

Q37. A cone floats with its axis horizontal in a liquid of density double its own. Fine the pressure on its base prove that the if $\theta$ be the inclination to the vertical of the resultant thrust on the curved surface and $\alpha$ the semi-vertical angle of the cone, then $\theta = \tan^{-1}\left[\frac{4}{\pi}\tan\alpha\right]$

**(Year 1998)**
**(20 Marks)**

Q38. A right circle cone of density $\rho$ floats just immersed with its vertex downwards in a vessel containing two liquids of densities $\sigma_1$ and $\sigma_2$ respectively. Show that the plane of separation of the two liquids cut off from the axis of the cone a fraction $\left[\frac{(\rho-\sigma_2)}{(\sigma_1-\sigma_2)}\right]^{\frac{1}{3}}$ of its length.

**(Year 1998)**
**(20 Marks)**

Q39. Two particles of masses $m_1$ and $m_2$ moving in coplanar parabolas round the sun collide at right coalesce when their common distance from the sun is $R$ show that the subsequent path of the combine particles is an ellipse of major axis $(m_1+m_2)^2\frac{R}{2m_1m_2}$.

**(Year 1998)**
**(20 Marks)**

Q40. One end of an inextensible string is fixed to a point O and to the other end is tied a particle of mass m the particle is projected from its position of equilibrium vertically below O with a horizontal velocity so as to Cary it right round the circle prove that the some of the tensions at the end of a diameter is constant.

**(Year 1998)**
**(20 Marks)**

Q41. Show how to cut of uniform cylinder a cone whose base coincides with that of a cylinder, so that the centre of gravity of the remaining solid may coincide with the vertex of the cone.

**(Year 1998)**
**(20 Marks)**

Q42. A heavy elastic string whose natural length is $2\pi a$ is placed round a smooth one whose axis is vertical and whose semi-vertical angle is $\alpha$ if $W$ be the weight and $\lambda$ the modulus of elasticity of the string prove that it be in equilibrium when in the form of a circle whose radius is $a\left(1+\frac{w}{2\pi\lambda}\cot\alpha\right)$.

**(Year 1998)**
**(20 Marks)**

Q43. Two lumps of clay each of rest mass $m_0$ collide head on with velocity $3/\,5c$ , and stick together. What is the mass of the composite lump?

**(Year 1999)**
**(20 Marks)**

Q44. Defined relativistic energy and momentum and establish $E^2 = p^2c^2 + m_0^2c^4$ usual notation.

**(Year 1999)**
**(20 Marks)**

Q45. If u and are two velocities in the same direction and V is their resultant velocity given by $\tanh^{-1}\frac{V}{c} = \tanh^{-1}\frac{u}{c} + \tanh^{-1}\frac{v}{c}$ then deduce the law composition of velocities from this equation.

**(Year 1999)**
**(20 Marks)**

Q46. Masses m and of two gasses in which the ration of the pressure of the density $\left(\frac{p}{\rho}\right)$ are respectively $k$ and $k'$, are mixed at the same temperature. Prove that the ration of the pressure to the density ion the compound is $\frac{mk+m'k'}{m+m'}$.

**(Year 1999)**
**(20 Marks)**

Q47. Drive the Lorentz transformation equations.

**(Year 1999)**
**(20 Marks)**

Q48. Two solids are each weighed in succession in three homogeneous liquid of different densities. If the weight of the one are $w_1, w_2, w_3$ and those of the other are $w_1, w_2$ and $w_3$ prove that

$$w_1(w_2 - w_3) + w_2(w_3 - w_1) + w_3(w_1 - w_2) = 0$$

**(Year 1999)**
**(20 Marks)**

Q49. An ellipse is just immersed in water (touching water surface) with its major axis vertical. Show that if the centre of pressure coincides with the focus the eccentricity of ellipse $\frac{1}{4}$

**(Year 1999)**
**(20 Marks)**

Q50. A particle of mass $m$ projected vertically under gravity, the resistance of air being $mk$ (velocity). Show that the greatest height attained by the particle is $V^2/g[\lambda - \log(1 + \lambda)]$ where $V$ is the terminal velocity of the particle and $\lambda V$ is the initial vertical velocity.

**(Year 1999)**
**(20 Marks)**

Q51. A particle move with a centre acceleration $\mu(r^5 - c^4 r)$ being projected from an apse at a distance $c$ with a velocity $\sqrt{\frac{2\mu}{3}}c^3$. Determine its path.

**(Year 1999)**
**(20 Marks)**

Q52. If in a simple harmonic motion $u, v, w$ be the velocities at a distance $a, b, c$ from a fixed point on the straight line (which is not the centre of force), Show that the period $T$ is given by the equation

$$\frac{4\pi^2}{T^2}(b - c)(c - a)(a - b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

**(Year 1999)**
**(20 Marks)**

Q53. A uniform chain, of length $l$ and weight $W$ hangs between two fixed points at the same level and weight $W'$ is attached at the middle point If $K$ be the sag in the middle prove that pull on either point of support is $\frac{K}{2l}W + \frac{l}{4K}W' + \frac{l}{8K}W$.

**(Year 1999)**
**(20 Marks)**

Q54. A string of length $a$ forms the shorted diagonal of a thrombus formed of four uniform rods, each of length $b$ and weight $W$, are hinged together. If one of the rods be supported in horizontal position, prove that the tension of the string is $\frac{2w(2b^2-a^2)}{b(4b^2-a^2)^{\frac{1}{2}}}$

**(Year 1999)**
**(20 Marks)**

Q55. A perfectly rough plane is inclined at an angle $\alpha$ to the horizon. Show that the least eccentricity of the ellipse which can rest on the plane is $\left[\frac{2\sin\alpha}{1+\sin\alpha}\right]^{\frac{1}{2}}$

**(Year 1999)**
**(20 Marks)**

Q56. A conical buoy 1 meter long, and of base diameter 1.2 meter, floats in water with its apex downwards. Determine the minimum weight of the buoy, for stable equilibrium.

**(Year 2000)**
**(15 Marks)**

Q57. A small bead is projected with any velocity along the smooth circle wire under the action of a force varying inversely as the fifth power of the distance from an centre of force situated on the circumference. Power that the pressure on the wire is constant.

**(Year 2000)**
**(15 Marks)**

Q58. Assuming that the earth attracts points inside it with a force which varies as the distance from its centre, show that if a straight frictionless airless tunnel be made from one point of the earth's surface to any point, a train would traverse the tunnel in slightly less than three quarter of an hour. Assume the earth to be a homogeneous sphere of radius 6400 km.

**(Year 2000)**
**(15 Marks)**

Q59. A telephone wire weighing 0.04 1b per food has a horizontal span of 150 feet and sag of 1.5 feet. Find the length of the wire and also find maximum tension.

**(Year 2000)**
**(15 Marks)**

Q60. A trapezoidal plate having its parallel sides of length $x$ and , $(x > y)$ at a distance $z$ apart, is immersed vertically in water into $x$ side uppermost (horizontal) at a depth $d$ below the water surface. Find the total thrust on the surface.

**(Year 2000)**
**(12 Marks)**

Q61. Prove that a central force motion is a motion in a plane and the areal velocity of a particle is constant.

**(Year 2000)**
**(12 Marks)**

Q62. If $T$ is the tension at any point $P$ of a catenary and $T_0$ that at the lowest point $C$ the show that $T^2 - T_0^2 = W^2$ where $W$ is the weight of the arc $CP$ of the catenary.

**(Year 2000)**
**(12 Marks)**

Q63. A right circular cylinder floating with its axis horizontal and in the surface is displaced in the vertical plane through the axis. Discuss its stability of equilibrium.

**(Year 2001)**
**(15 Marks)**

Q64. $OA, OB$ And $OC$ are edges of a side a, and $OO', AA', BB'$ and $CC'$ are it diagonals. Along $OB', O'A, BC$ and $C'A'$ act force equal to $P, 2P, 3P$ and $4P$ respectively. Reduce the system to a force at $O$ together with a couple.

**(Year 2001)**
**(15 Marks)**

Q65. A particle of mass $M$ is at rest and begins to move under the action of a constant force $F$ in a fixed direction It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity $V$ which deposits matter on it at a constant rate $\rho$ show that the mass of the particle will be m when it has travelled a distance $\frac{k}{\rho^2}\left[m - M\left\{1 + \log_e \frac{m}{M}\right\}\right]$ where $K = F - \rho V$.

**(Year 2001)**
**(15 Marks)**

Q66. A comet describing a parabola under inverse square law about the sun, when nearest to it suddenly breaks up, without gain or loss of kinetic energy , into two equal portions one of which describes a circle. Prove that that the other will describe a hyperbola of eccentricity 2.

**(Year 2001)**
**(15 Marks)**

Q67. The middle points of opposite sides of a jointed quadrilateral are connected by light rods of length $l, l'$. if $T, T'$ be the tension in these rods, prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0$$

**(Year 2001)**
**(12 Marks)**

Q68. A solid right circular cone with semi-vertical angle $\alpha$ is just immersed in a liquid with a generating line on the surface if $\theta$ be the inclination of the vertical with the resultant thrust on the curve surface, prove that

$$(1 - 3\sin^2\alpha)\tan\theta = 3\sin\alpha\cos\alpha$$

**(Year 2001)**
**(12 Marks)**

Q69. Find the law of force to the pole when the path of a particle is the cardioid $r = a(1-\cos\theta)$ and prove that if $F$ be the force at the sphere and $v$ the velocity there then $3v^2 = 4aF$

**(Year 2001)**
**(12 Marks)**

Q70. A solid cylinder floats in a liquid with its axis vertical. Let $\sigma$ be the radius of the specific gravity of the cylinder to the of the liquid. Prove that the equilibrium stable if the ratio of the radius of the base to the height is greater than $\sqrt{2\sigma(1-\sigma)}$.

**(Year 2002)**
**(15 Marks)**

Q71. Five weightless rods of equal length are jointed together so as to from a rhombus $ABCD$ with a diagonal $BD$. If a weight $W$ be attached to $C$ and the system be suspended from a point $A$, show that the thrust in $BD$ is equal to $\frac{W}{\sqrt{3}}$

**(Year 2002)**
**(15 Marks)**

Q72. A particle describes a curve with constant velocity and its angular velocity about a given point O varies inversely as its distance from O. show that the curve is an equiangular spiral.

**(Year 2002)**
**(15 Marks)**

Q73. A heavy particle of mass m slides on a smooth arc of a cycloid in a medium whose resistance is $v$, being the velocity of the particle and $c$ being the distance of the starting point from the vertex upwards, find the velocity of the particle at the cusp.

**(Year 2002)**
**(15 Marks)**

Q74. Half of the ellipse is vertically immersed in water with minor axis just in the surface. Find the surface the position of centre of pressure.

**(Year 2002)**
**(12 Marks)**

Q75. Obtain the equation of the curve in which a string hangs under gravity from two fixed points (not lying in a vertical line) when line mass density at each of its points varies as the radius of curvature of the curve.

**(Year 2002)**
**(12 Marks)**

Q76. A particle, whose mass in m is acted upon by a force $m\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it stats from rest at a distance a, from the origin. Show that the time taken by it to reach the it origin is $\frac{\pi}{4}$

**(Year 2002)**
**(12 Marks)**

Q77. An ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ immersed vertically in a fluid with its semi-axis of length a horizontal. If its centre be at a depth $h$, finds the depth of the centre of pressure.

**(Year 2003)**
**(15 Marks)**

Q78. A ladder on a horizontal floor lean against a vertical wall the coefficients of friction of the floor and the wall with the ladder are $\mu$ and $\mu$ respectively If a man, whose weight is $n$ times that of the ladder, wants to climb up the ladder, find the minimum safe angle of the ladder with the floor.

**(Year 2003)**
**(15 Marks)**

Q79. If a particle slides down a smooth cycloid, starting from a point whose actual distance from the vertex is $b$, prove that its speed at any time $t$ is $\frac{2\times b}{T}\sin\left(\frac{2xt}{T}\right)$ where $T$ is the time of complete oscillation of the particle.

**(Year 2003)**
**(15 Marks)**

Q80. A particle describes the curve $r=a(1+\cosh\theta)\ /(\cosh\theta-2)$ under a force F to the pole. Show that the law of force is $F\propto\frac{1}{r^4}$

**(Year 2003)**
**(12 Marks)**

Q81. A sphere of weight W and radius a lies within a fixed spherical shell of radius b . A particle of weight W is fixed to the upper end of the vertical diameter. Prove that equilibrium is stable if $\frac{W}{w}>\frac{b-2a}{a}$

**(Year 2003)**
**(12 Marks)**

Q82. An elastic string of natural length $a+b$,where $a>b$ and modulus of elasticity $\lambda$ has particle of mass m attached to it at a distance a form one end which is fixed to a point A of a smooth horizontal plane. The other end of the string is fixed to a point B so that the string is just unstretched. If the particle be held at B and then released find the periodic time and the distance in which the particle will oscillate to and fro.

**(Year 2003)**
**(15 Marks)**

Q83. A cone, of given weight and volume, floats with its vertex downwards. Prove that the surface of the cone in contact with the liquid is least when its vertical angle is $2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

**(Year 2004)**
**(15 Marks)**

Q84. A uniform bar $AB$ weights $12N$ and rests with one part $AC$ of length $8m$, on a horizontal table and the remaining part $CB$ projecting over the edge of the table if the bar is on the point of overbalancing when a weight of $5N$ is placed on it at point $2m$ from $A$ and a weight of $7\ N$ is hung from $B$ find the length of $AB$.

**(Year 2004)**
**(15 Marks)**

Q85. A car of mass 750 kg is running up a hill of 1 in 30 at a steady speed of 36 km/hr; the friction is equal to the weight of 40 kg. Find the work done in 1 second.

**(Year 2004)**
**(15 Marks)**

Q86. Prove that the velocity required to project a particle from a height h to fall at a horizontal distance a from a point of projection is at least equal to $\sqrt{g[\sqrt{a^2+h^2}-h]}$

**(Year 2004)**
**(15 Marks)**

Q87. A circular area of radius $a$ is immersed with its plane vertical and its centre at a depth $c$. Find position of its centre of pressure.

**(Year 2004)**
**(12 Marks)**

Q88. A non uniform string hangs under gravity. Its cross-section at any point is inversely proportion to the tension at that point. Prove that the curve in which the string hangs is an arc of a parabola with its axis vertical.

**(Year 2004)**
**(12 Marks)**

Q89. A point moving with uniform acceleration describes distances $s_1$ and $s_2$ metres in successive intervals of time $t_1$ and $t_2$ seconds. Express the acceleration in terms of $s_1, s_2, t_1$ and $t_2$.

**(Year 2004)**
**(12 Marks)**

Q90. A rectangular lamina of length $2a$ and breadth $2b$ is completely immersed in a vertical plane, in a fluid so that its centre is at a depth $h$ and the sides $2a$ make an angle $\alpha$ with the horizontal. Find the position of the centre of pressure.

**(Year 2005)**
**(15 Marks)**

Q91. Two equal uniform rods $AB$ and $AC$ of length a each are freely joined at $A$, and are placed symmetrically over two smooth pegs on the same horizontal level at a distance $c$ apart $(3c < 2a)$. A weight equal to the of a road, is suspended from the joint $A$ in the position of equilibrium, find the inclination of the either rod with the horizontal by the principal of virtual work.

**(Year 2005)**
**(15 Marks)**

Q92. Two particles connected by a fine string are constrained to move in a fine cycloid tube in a verticals plane. The axis of the cycloid is vertical with vertex upwards. Prove that tension in the sting is constant throughout the motion.

**(Year 2005)**
**(15 Marks)**

Q93. A particle is projected along the inner side of a smooth vertical of radius a so that its velocity particle at the lowest point is $u$. Show that if $2ag < u^2 < 5ag$ the particle and will leave the circle before arriving at the highest point and will describe a parabola whose latus rectum is $\frac{2(u^2-2ga)^3}{27g^3a^2}$

**(Year 2005)**
**(15 Marks)**

Q94. If a number of concurrent forces be represented in magnitude and direction by the sides of a closed polygon, taken in other then show that these force are in equilibrium.

**(Year 2005)**
**(12 Marks)**

Q95. A body of mass $(m_1 + m_2)$ moving in a straight line is split into two parts of masses $m_1$ and $m_2$ by an internal explosion which generates kinetic energy $E$. If after the explosion, the two parts move in the same line as before, find their relative velocity.

**(Year 2005)**
**(12 Marks)**

Q96. A uniform rod of length $2a$ can terms freely about one end which is fixed at a height $h(< 2a)$ above the surface of the liquid if the densities of thee rod and liquid be $\rho$ and $\sigma$, show that the rod can rest either in a vertical position or inclined at an angle $\theta$ to the vertical such that $\cos\theta = \frac{h}{2a}\sqrt{\frac{\sigma}{\rho-\sigma}}$

**(Year 2006)**
**(15 Marks)**

Q97. Show that the length of an endless chain which will hang over a circular pulley of radius $C$ so as to be in contact with two third of the circumference of the pulley is $C\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$.

**(Year 2006)**
**(15 Marks)**

Q98. If $u$ and $v$ are the velocity of projected and the terminal velocity respectively of a particle rising vertically against a resistance varying as the square of the velocity prove that the time taken by the particle to reach the highest point is $\frac{v}{g}\tan^{-1}\left(\frac{u}{v}\right)$

**(Year 2006)**
**(15 Marks)**

Q99. A particle, whose mass in $m$ is acted upon by a force $m\left(x + \frac{a^4}{x^3}\right)$ towards the origin If it starts from rest at a distance a, show that it will arrive at origin in time $\frac{\pi}{4}$

**(Year 2006)**
**(15 Marks)**

Q100. Find the depth of the centre of pressure of a triangular lamina with a vertex I in the surface of the liquid and other two vertices at depths b and c from the surface.

**(Year 2006)**
**(12 Marks)**

Q101. A particle is free to move on a smooth vertical circular wire of radius a. it is projected horizontal from the lowest pint with velocity $2\sqrt{ga}$ . Show that the reaction between the particle and the wire is zero after is time

$$\sqrt{\frac{a}{g}}\log(\sqrt{5} + \sqrt{6}).$$

**(Year 2006)**
**(12 Marks)**

Q102. The middle points of opposite sides of a jointed quadrilateral are connected by light rods of length $l, l'$. if $T, T'$ be the tension in these rods, prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0$$

**(Year 2006)**
**(12 Marks)**

Q103. A cone whose vertical angle is $\frac{\pi}{3}$ has its lowest generator horizontal and is filled with a liquid. Prove that the pressure on the curved surface $\frac{W}{2}\sqrt{19}$ is where $W$ the weight of the liquid is.

**(Year 2007)**
**(15 Marks)**

Q104. A particle attached to a fixed peg O by a string of length $l$ is lifted up with the string horizontal and then let go. Prove that when the string makes an angle $\theta$ with the horizontal the resultant acceleration is $g\sqrt{(1 + 3\sin^2\theta)}$.

**(Year 2007)**
**(15 Marks)**

Q105. A particle is performing simple harmonic motion of period $T$ about a centre $O$. It passes through a point $P(OP = P)$with velocity $v$ in the direction $OP$. show that the time which elapses before it returns to $P$ is $\frac{T}{\pi}\tan^{-1}\frac{uT}{2\pi p}$

**(Year 2007)**
**(15 Marks)**

Q106. A quadrant of the ellipse $x^2 + 4y^2 = 4$ is just immersed vertically in a homogeneous liquid with the major axis in the surface. Find the centre of pressure.

**(Year 2007)**
**(12 Marks)**

Q107. A particle falls from rest under gravity in a medium whose resistance varies as the velocity of the particle. Find the distance fallen by the particle and its velocity at time $t$.

**(Year 2007)**
**(12 Marks)**

Q108. A uniform string of length one meter hangs over two smooth pegs P and Q at different heights. The parts which hand vertical are of length 34 cm and 26 cm. find the ratio in which the vertex of the catenary divides the whole string.

**(Year 2007)**
**(12 Marks)**

Q109. A uniform beam of length $l$ rests with its ends on two smooth planes which intersect in a horizontal line if the inclination of the planes to the horizontal are $\alpha$ and $\beta(\beta > \alpha)$ show that the inclination $\theta$ of the beam to the horizontal, in one of the equilibrium positions, is given by $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$ and show that the beam is unstable in this position.

**(Year 2007)**
**(15 Marks)**

Q110. A solid right circle cone whose height is h and radius of whose base is r, is placed on an inclined plane it is presold right circle cone whose height is h and radius of whose base is r, is placed on an inclined plane it prevented from sliding. If the inclination $\theta$ of the plane (to the horizontal) be gradually decreased find when the cone will topple over. For a cone whose semi- vertical angle is $30°$ determine the circular value of $\theta$ which when exceeded, the cone will topple over.

**(Year 2008)**
**(15 Marks)**

Q111. A ladder of weight 10 kg. rests on a smooth horizontal ground leaning against a smooth vertical wall at an inclination $\tan^{-1} 2$ with the horizontal and is prevented from slipping by a string attached at it lower end and to the junction of the floor and the wall A body of weight 30 kg begins to ascend the ladder. If the string can bear a tension of 10 kg. wt., how far along the ladder cans the boy rise with safety?

**(Year 2008)**
**(15 Marks)**

Q112. A shell lying in a strength smooth horizontal tube suddenly breaks into two portions of masses $m_1$ and $m_2$ if $s$ be the distance between the two masses inside the tube after time $t$, show that the work done by the explosion can be written as equal to $\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\frac{s^2}{t^2}$.

**(Year 2008)**
**(15 Marks)**

Q113. A particle of mass $m$ moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}, u = \frac{1}{r}$, $a > b$ , $a, b$ and $\mu(> 0)$ being given constants. It is projected from an apse at a distance $a + b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is given by the equation $r = a + b\cos\theta$, where $(r, \theta)$ are the plane polar coordinates of a point.

**(Year 2008)**
**(15 Marks)**

Q114. A particle $P$ moves in a plane such that it is acted on by two constant velocities u and v respectively along the direction $OX$ and along the direction perpendicular to $OP$ where $O$ is same fixed point, the is the origin. Show that the path traversed by $P$ is a conic section with focus at $O$ and eccentricity $\frac{u}{v}$

**(Year 2008)**
**(15 Marks)**

Q115. A straight uniform beam of length '$2h$' rests in limiting equilibrium in contact with a rough vertical wall of height '$h$' with one end on a rough horizontal plane and with the other end projecting beyond the wall If both the wall and the plane be equally rough, prove that '$\lambda$' the angle of friction, is given by $\sin 2\lambda = \sin\alpha \sin 2\alpha,$ '$\alpha$' being the inclination of the beam to the horizon.

**(Year 2008)**
**(12 Marks)**

Q116. A smooth parabolic tube is placed with vertex downwards in vertical plane A particle slides down the tube from rest under the influence of gravity prove that in any position, the reaction of the tube is equal to $2w\left(\frac{h+a}{\rho}\right)$, where '$w$' is the weight of the particle '$p$' the radius of curvature of the tube '$4a$' its latus rectum and '$h$' the initial vertical height of the particle above the vertex of the tube.

**(Year 2008)**
**(12 Marks)**

Q117. Find the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with three-fourth of the circumference of the pulley.

**(Year 2009)**
**(15 Marks)**

Q118. A particle is projected with velocity $V$ from the cusp of a smooth inverted cycloid down are arc. Show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\cot^{-1}\left(\frac{V}{2\sqrt{ag}}\right)$

**(Year 2009)**
**(10 Marks)**

Q119. A body is describing an ellipse of eccentricity e under the action of a central force directed towards a focus and when at the nearer apse, the cente of force is transferred to the other focus. Find the eccentricity of the new orbit in terms of the eccentricity of the original orbit.

**(Year 2009)**
**(12 Marks)**

Q120. One end of light elastic of natural length l and modulus of elasticity 2mg is attached to fixed point O and the other end to a particle of mass m, the particle initially held at rest at O is let fall. Find the greatest extension of the string during the motion and show that the particle will reach O again after a time $(\pi + 2 - \tan^{-1} 2)\sqrt{\frac{21}{g}}$

**(Year 2009)**
**(20 Marks)**

Q121. A shot fired with a velocity V at an elevation $\alpha$ strikes a point $P$ in a horizontal plane through the point of projection. If the point $P$ is receding from the gun with velocity $v$, show that the elevation must be changed to $\theta$ where

$$\sin 2\theta = \sin 2\alpha + \frac{2v}{V}\sin\theta.$$

**(Year 2009)**
**(12 Marks)**

Q122. A uniform rod $AB$ is movable about a hinge at $A$ and rests with one end in contact with a smooth vertical wall. If the rod is inclined at an angle of $30°$ with the horizontal, find the reaction at the hinge in magnitude and direction.

**(Year 2009)**
**(12 Marks)**

Q123. Solid hemisphere is supported by a string fixed to point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If $\theta$ and $\phi$ are the inclination of the string and the plane base of the hemisphere to the vertical, prove by using the principal of virtual work that $\tan\phi = \frac{3}{8} + \tan\theta$

**(Year 2010)**
**(20 Marks)**

Q124. A particle moves with a central acceleration $\mu(r^5 - 9r)$ being projected from an apse at a distance $\sqrt{3}$ with velocity $3\sqrt{2\mu}$ . Show that the path is the curve $x^4 + y^4 = 9$

**(Year 2010)**
**(20 Marks)**

Q125. A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half.

**(Year 2010)**
**(20 Marks)**

Q126. If $v_1$ , $v_2$ , $v_3$ are the velocities at three points $A, B, C$ of the path of a projectile, where the inclinations to the horizon are $\alpha, \alpha - \beta, \alpha - 2\beta$ and if $t_1, t_2$ are the times of describing the arcs $AB, BC$ respectively. Prove that $v_3 t_1 = v_1 t_2$ and $\frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$

**(Year 2010)**
**(12 Marks)**

Q127. A particle of mass $m$ moves on straight line under an attractive force $mn^2x$ towards a point O on the line, where $x$ is the distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t = 0$, find $x(t)$ for any time $t > 0$.

**(Year 2011)**
**(10 Marks)**

Q128. After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half its velocity if it now reaches the ground in 1 second, find the height of glass above the ground.

**(Year 2011)**
**(10 Marks)**

Q129. A ladder of weight W rests with one end against a smooth vertical wall and the other end rest on a smooth floor. If the inclination of the ladder to the horizon is $60°$, find the horizontal force that they must be a applied to the lower end to prevent the ladder from slipping down.

**(Year 2011)**
**(20 Marks)**

Q130. A mass of 560kg. moving with a velocity of 240 m/ sec strikes a fixed target and is brought to rest in $\frac{1}{100}$ sec. Find the impulse of the blow on the target and assuming the resistance to be uniform through out the time taken by the body in coming to rest, find the distance through which it penetrates.

**(Year 2011)**
**(20 Marks)**

Q131. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a meter short of it when the angle of projection is $\alpha$ and goes $y$ meter beyond $w$ then the angle of projection is $\beta$. If the velocity of projection is assumed same in all cases, find the correct angle of projection.

**(Year 2011)**
**(10 Marks)**

Q132. The velocity of a train increases from 0 to $v$ at constant acceleration $f_1$ then remains constant for an interval and again decreases to 0 at a constant retardation $f_2$, If the total distance described is $x$ find the total time taken.

**(Year 2011)**
**(10 Marks)**

Q133. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log\left[\frac{1+\sqrt{1+\mu^2}}{\mu}\right]$ where $\mu$ the coefficient of friction is.

**(Year 2012)**
**(20 Marks)**

Q134. A heavy hemispherical shell of radius $a$ has a particle attached a point on the rim, and rests with the curved surface in contact with a rough sphere of radius b at the highest point. Prove that it $\frac{b}{a} > \sqrt{5} - 1$ the equilibrium is stable, whatever be the weight of the particle.

**(Year 2012)**
**(20 Marks)**

Q135. A heavy ring of mass $m$, slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distance a from the rod and has a mass $M(> m)$ fastened to its order end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley it will descend a distance $\frac{2Mma}{M^2-m^2}$ before coming to rest.

**(Year 2012)**
**(20 Marks)**

Q136. A particle moves with an acceleration $\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a from the origin, find its velocity when its distance from the origin is $\frac{a}{2}$.

**(Year 2012)**
**(12 Marks)**

Q137. A uniform ladder rests at an angle of $45°$ with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If $\mu$ and $\mu'$ are the coefficients of limiting friction between the ladder and the ground and wall respectively, the find the minimum horizontal force required to move the lower end of the ladder towards the wall.

**(Year 2013)**
**(15 Marks)**

Q138. Six equal rods AB, BC, CD, DE, EF and FA are each of weight w and are freely jointed at their extremities so as to from a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by string. Find the tension in the string.

**(Year 2013)**
**(15 Marks)**

Q139. A particle of mass 2.5 kg hangs at the end of a string 0.9 m long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity 8 m/sec. Find the velocity of the particle and tension in the string when the string is (i) horizontal (ii) vertically upward.

**(Year 2013)**
**(20 Marks)**

Q140. The base of an inclined plane is 4 metres in length and the height is 3 metres. A force of 8kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction between the plane and the weight.

**(Year 2013)**
**(10 Marks)**

Q141. A body is performing S.H.M in a straight line $OPQ$. Its velocity is zero at points $P$ and $Q$ whose distances from $O$ are $x$ and $y$ respectively and its velocity is $v$ at the mid-point between $P$ and $Q$. Find the time of one complete oscillation.

**(Year 2013)**
**(10 Marks)**

Q142. A particle is performing a simple harmonic motion (S.H.M) of period $T$ about a centre $O$ with amplitude $a$ and it passes through a points $P$ where $OP = b$ in the direction $OP$ . Prove that the time which elapse before it returns to $P$ is $\frac{T}{\pi}\cos^{-1}\left(\frac{b}{a}\right)$.

**(Year 2014)**
**(10 Marks)**

Q143. A regular pentagon ABCDE, formed of equal heavy uniform bars jointed together is suspended from the joint A, and is maintained in from by a light road joining the middle points of BC and DE. Find the stress in this rod.

**(Year 2014)**
**(15 Marks)**

Q144. Two equal uniform rods AB and AC, each of length $l$ are freely jointed at A and rest on a smooth fixed vertical circle of radius $r$. If $2\theta$ is the angle between the roads, then find the relation between $l, r$ and $\theta$ by using the principal of virtual work.

**(Year 2014)**
**(10 Marks)**

Q145. A particle is acted on by a force parallel to the axis of $y$ whose acceleration (always towards the axis of $x$) is $\mu y^{-2}$ and when $y = a$ it is projected parallel to the axis of $x$ with velocity $\sqrt{\frac{2\mu}{a}}$. Find the parametric equation on the path of the particle. Here $\mu$ is constant.

**(Year 2014)**
**(15 Marks)**

Q146. A particle of mass $m$, hanging vertically from a fixed point by a light inextensible cord of length $l$ is struck by a horizontal blow which imparts of it a velocity $2\sqrt{gl}$ . Find the velocity and height of the particle from of its initial position when the cord becomes slack.

**(Year 2014)**
**(15 Marks)**

Q147. Find the length of an endless chain which will hang over circular pulley of radius 'a' so as to be in contact with the two thirds of the circumference of the pulley.
A particle moves in a plane a force towards a fixed centre proportional to the distance. If the path of the particle has apsidal distance a, b (a> b), then find the equation of the path.

**(Year 2015)**
**(12+13 Marks)**

Q148. A particle is projected from the base of a bill whose slope is that of a right circular cone whose axis is vertical. The projectile grazes the vertex and strikes the hill again at point on the base. If the semi vertical angle of the cone is $30°$ h is height determine the initial velocity u of the projection and its angle of projection.

**(Year 2015)**
**(13 Marks)**

Q149. A mass starts from rest at a distance 'a' from the center of force which attract inversely as the distance find the time of arriving at the center.

**(Year 2015)**
**(13 Marks)**

Q150. Two equal ladders of weight 4kg each are placed so as to lean at A against each other with their end resting on a rough floor, given the coefficient of friction is $\mu$. The ladders at A make an angle $60°$ with each other. find what weight on the top would cause them to slip.

**(Year 2015)**
**(13 Marks)**

Q151. A rod of 8kg is movable in a vertical plane about a hinge at one end another end is fastened a weight equal to half of the rod, this is fastened by a string of length $l$ to a point at a height to above the hinge vertically. Obtain the tension in the sting.

**(Year 2015)**
**(10 Marks)**

Q152. A body moving under SHM has an amplitude $a'$ and time period $T'$ If velocity is trebled, when the distance from mean position is $\frac{2}{3}a$, the period being unaltered find the new amplitude.

**(Year 2015)**
**(10 Marks)**

Q153. A particle moves in a straight line. Its acceleration is directed towards a fixed point $O$ in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{\frac{1}{3}}$ when it is at a distance $x$ from $O$. If it starts rest at a distance $a$ form $O$, then find the time the particle will arrive at $O$.

**(Year 2016)**
**(15 Marks)**

Q154. A square $ABCD$ the length of whose side is a is fixed in a vertical plane with two of its sides horizontal An endless string of length $l(> 4a)$ passes over four pegs at the angle of the board and through a ring of weight $W$ which is hanging vertically show that the tension of the string is $\frac{W(l-3a)}{\sqrt{l^2-6la+8a^2}}$.

**(Year 2016)**
**(20 Marks)**

Q155. Two weights $P$ and $Q$ are suspended from a fixed point $O$ by strings $OA$, $OB$ and are kept apart by a light rod $AB$ if the strings $OA$ and $OB$ make angle $\alpha$ and $\beta$ with the rod $AB$ show that the angle $\theta$ which the rod makes with the vertical is given by $\theta = \frac{P+Q}{P\cot\alpha - Q\cot\beta}$

**(Year 2016)**
**(15 Marks)**

Q156. A uniform rod $AB$ of length $2a$ movable about a hinge at $A$ rests with other end against a smooth vertical wall. If $\alpha$ is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is $\frac{1}{2}W\sqrt{4+\tan^2\alpha}$ where $W$ is the weight of the rod.

**(Year 2016)**
**(15 Marks)**

Q157. A particle moves with a central acceleration, which varies inversely as the cube of the distance. If it is projected from an apse at a distance $a$ from the origin with a velocity which $\sqrt{2}$ is times the velocity for a circle or radius $a$ then find the equation to the path.

**(Year 2016)**
**(10 Marks)**

Q158. A spherical shot of $W$ gm weight and radius $r$ cm, lies at the bottom of cylindrical bucket of radius $R$ cm. The bucket is filled with water up to a depth of $h$ cm $(h > 2r)$. Show that the minimum amount of work done in lifting the shot just clear of the water must be $\left[W\left(h - \frac{4r^3}{3R^2}\right) + W'\left(r - h + \frac{4r^3}{3R^2}\right)\right]$ cm gm. $W'$ gm is the weight of water displaced by the shot.

**(Year 2017)**
**(16 Marks)**

Q159. A particle is free to move on a smooth vertical circular wire of radius $a$. At time $t = 0$ is projected along the circle from its lowest point $A$ with velocity just sufficient to carry it to the highest point $B$. Find the time $T$ at which the reaction between the particle and the wire is zero.

**(Year 2017)**
**(17 Marks)**

Q160. A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle $\phi$ with its curve surface touching the plane. Find the greatest admissible value of the inclination $\phi$ for equilibrium. If $\phi$ be less than this value, is the equilibrium stable?

**(Year 2017)**
**(17 Marks)**

Q161. A fixed wire is in the shape of the Cardioid$r = a(1 + \cos\theta)$, the initial line being the downward vertical. A small ring of mass $m$ can slide on the wire and is attached to the point $r = 0$ of the Cardioid by an elastic string of natural length $a$ and modulus of elasticity 4 $mg$. The string is released from rest when the string is horizontal. Show by using the laws of conservation of energy that $a\theta^2(1 + \cos\theta) - g\cos\theta\,(1 - \cos\theta) = 0, g$ being the acceleration due to gravity.

**(Year 2017)**
**(10 Marks)**

Q162. Suppose that the streamlines of the fluid flow are given by a family of curves $xy = c$. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines.

**(Year 2017)**
**(10 Marks)**

Q163. A particle projected from a given point on the ground just clears a wall of height $h$ at a distance $d$ from the point of projection. If the particle moves in a vertical plane and if the horizontal range is R, find the elevation of the projection.

**(Year 2018)**
**(10 Marks)**

Q164. A particle moving with simple harmonic motion in a straight line has velocities $v_1$ and $v_2$ at distances $x_1$ and $x_2$ respectively from the centre of its path. Find the period of its motion.

**(Year 2018)**
**(12 Marks)**

Q165. One end of a heavy uniform rod $AB$ can slide along a rough horizontal rod $AC$, to which it is attached by a ring $B$ and $C$ are joined by a string. When the rod is on the point of sliding, then $AC^2 - AB^2 = BC^2$. If $\theta$ is the angle between $AB$ and the horizontal line, then prove that the coefficient of friction is $\frac{\cot\theta}{2+\cot^2\theta}$.

**(Year 2019)**
**(10 Marks)**

Q166. The force of attraction of a particle by the earth is inversely proportional to the square of its distance from the earth's center. A particle, whose weight on the surface of the earth is $W$, falls to the surface of the earth form a height $3h$ above it. Show that the magnitude of work done by the earth's attraction force is $\frac{3}{4}\, hW$, where $h$ is the radius of the earth.

**(Year 2019)**
**(10 Marks)**

Q167. A body consists of a cone and underlying hemisphere. The base of the cone and the top of the hemisphere have same radius $a$. The whole body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}a$.

**(Year 2019)**
**(15 Marks)**

Q168. Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (star) and is equal to $\frac{\mu}{(dislance)^2}$, is a conic section. Find the conditions under which the path becomes $(i)$ ellipse, $(ii)$ parabola and $(iii)$ hyperbola.

**(Year 2019)**
**(15 Marks)**

Q169. A particle moving along the $y$-axis has an acceleration $Fy$ towards the origin, where $F$ is a positive and even function of $y$. The periodic time, when the particle vibrates between $y = -a$ and $y = a$, is $T$. Show that $\frac{2\pi}{\sqrt{F_1}} < T < \frac{2\pi}{\sqrt{F_2}}$ where $F_1$ and $F_2$ and the greatest and the least values of $F$ within the range $[-a, a]$. Further, show that when a simple pendulum of length $l$ oscillates through $30°$ on either side of the vertical line, $T$ lies between $2\pi\sqrt{l/g}$ and $2\pi\sqrt{l/g}\sqrt{\pi/3}$

**(Year 2019)**
**(20 Marks)**

## CIVIL SERVICES

## PREVIOUS YEAR QUESTIONS

## SEGMENT- WISE

# STATICS and DYNAMICS

1. A wire of length $l$ is cut into two parts which are bent in the form of a square and a circle respectively. Using Lagrange's method of undetermined multipliers, find the least value of the sum of the areas so formed. **[2022][15]**
2. A body of weight $w$ rests on a rough inclined plane of inclination $\theta$, the coefficient of friction, $\mu$, being greater than $\tan\theta$. Find the work done in slowly dragging the body a distance $'b'$ up the plane and then dragging it back to the starting point, the applied force being in each case parallel to the plane. **[2022][10]**
3. A projectile is fired from a point $\mathbf{O}$ with velocity $\sqrt{2gh}$ and hits a tangent at the point $P(x, y)$ in the plane, the axes OX and OY being horizontal and vertically downward lines through the point $O$, respectively. Show that if the two possible directions of projection be at right angles, then $x^2 = 2$ hy and then one of the possible directions of projection bisects the angle $POX$. **[2022][10]**
4. A cable of weight w per unit length and length $2l$ hangs from two points $\mathbf{P}$ and $\mathbf{Q}$ in the same horizontal line. Show that the span of the cable is $2l\left(1-\frac{2h^2}{3l^2}\right)$, where $\mathbf{h}$ is the sag in the middle of the tightly stretched position. **[2022][20]**
5. Suppose a cylinder of any cross-section is balanced on another fixed cylinder, the contact of curved surfaces being rough and the common tangent line horizontal. Let $\rho$ and $\rho'$ be the radii of curvature of the two cylinders at the point of contact and $h$ be the height of centre of gravity of the upper cylinder above the point of contact. Show that the upper cylinder is balanced in stable equilibrium if $h < \frac{\rho\rho'}{\rho+\rho'}$. **[2022][15]**
6. A chain of $n$ equal uniform rods is smoothly jointed together and suspended from its one end $A_1$. A horizontal force $\vec{P}$ is applied to the other end $A_{n+1}$ of the chain Find the inclinations of the rods to the downward vertical line in the equilibrium configuration. **[2022][15]**
7. Two rods $LM$ and $MN$ are joined rigidly at the point $M$ such that $(LM)^2 + (MN)^2 = (LN)^2$ and they are hanged freely in equilibrium from a fixed point $L$. Let $\omega$ be the weight per unit length of both the rods which are uniform. Determine the angle, which the rod $LM$ makes with the vertical direction, in terms of lengths of the rods. **[2021][10]**
8. If a planet, which revolves around the Sun in a circular orbit, is suddenly stopped in its orbit, then find the time in which it would fall into the Sun. Also, find the ratio of its falling time to the period of revolution of the planet. **[2021][10]**
9. A heavy string, which is not of uniform density, is hung up from two points. Let $T_1, T_2, T_3$ be the tensions at the intermediate points $A, B, C$ of the catenary respectively where its inclinations to the horizontal are in arithmetic progression with common difference $\beta$. Let $\omega_1$ and $\omega_2$ be the weights of the parts $AB$ and $BC$ of the string respectively. Prove that
   **(i)** Harmonic mean of $T_1, T_2$ and $T_3 = \frac{3T_2}{1+2\cos\beta}$
   **(ii)** $\frac{T_1}{T_3} = \frac{\omega_1}{\omega_2}$ **[2021][20]**

**10.** A heavy particle hangs by an inextensible string of length $\boldsymbol{a}$ from a fixed point and is then projected horizontally with a velocity $\sqrt{2gh}$. If $\frac{5a}{2} > h > a$, then prove that the circular motion ceases when the particle has reached the height $\frac{1}{3}(a+2h)$ from the point of projection. Also, prove that the greatest height ever reached by the particle above the point of projection is $\frac{(4a-h)(a+2h)^2}{27a^2}$ **[2021][15]**

**11.** Describe the motion and path of a particle of mass $\boldsymbol{m}$ which is projected in a vertical plane through a point of projection with velocity $\boldsymbol{u}$ in a direction making an angle $\boldsymbol{\theta}$ with the horizontal direction. Further, if particles are projected from that point in the same vertical plane with velocity $4\sqrt{g}$, then determine the locus of vertices of their paths **[2021][15]**

**12.** A uniform rod, in vertical position, can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of the rod and equal to half its weight. At what inclination to the vertical will the rod rest**?** **[2020][10]**

**13.** A light rigid rod $\boldsymbol{ABC}$ has three particles each of mass $\boldsymbol{m}$ attached to it at $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$. The rod is struck by a blow $\boldsymbol{P}$ at right angles to it at a point distant from $\boldsymbol{A}$ equal to $\boldsymbol{BC}$ Prove that the kinetic energy set up is $\frac{1}{2}\frac{P^2}{m}\frac{a^2-ab+b^2}{a^2+ab+b^2}$ where $AB = a$ and $BC = b$. **[2020][10]**

**14.** A beam $\boldsymbol{AD}$ rest on two supports $\boldsymbol{B}$ and $\boldsymbol{C}$, where $AB = BC = CD$. It is found that the beam will tilt when a weight of $\boldsymbol{p}$ kg is hung from $\boldsymbol{A}$ or when a weight of $\boldsymbol{q}$ kg is hung from $\boldsymbol{D}$. Find the weight of the beam**.** **[2020][15]**

**15.** A square framework formed of uniform heavy rods of equal weight $\boldsymbol{W}$ jointed together, is hung up by one corner. A weight $\boldsymbol{W}$ is suspended from each of the three lower corners and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod. **[2020][10]**

**16.** A particle starts at a great distance with velocity $\boldsymbol{V}$. Let $\boldsymbol{p}$ be the length of the perpendicular from the centre of a star on the tangent to the initial path of the particle. Show that the least distance of the particle from the centre of the star is $\boldsymbol{\lambda}$, where $V^2\lambda = \sqrt{\mu^2 + p^2V^4} - \mu$. Here $\boldsymbol{\mu}$ is a constant **[2020][10]**

**17.** A four-wheeled railway truck has a total mass $\boldsymbol{M}$, the mass and radius of gyration of each pair of wheels and axle are $\boldsymbol{m}$ and $\boldsymbol{k}$ respectively, and the radius of each wheel is $\boldsymbol{r}$ Prove that if the truck is propelled along a level track by a force $\boldsymbol{P}$, the acceleration is $\frac{P}{M+\frac{2mk^2}{r^2}}$, and find the horizontal force exerted on each axle by the truck. The axle friction and wind resistance are to be neglected. **[2020][15]**